

# Chromomagnetism in nuclear matter

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**Abstract** Quarks are color charged particles. Due to their motion there is a strong possibility of generation of color magnetic field. It is shown that however hadrons are color singlet particles they may have non-zero color magnetic moment. Due to this color magnetic moment hadrons can show color interaction. In this paper we have studied the chromomagnetic properties of nuclear matter.

**Keywords** Color magnetic moment, Non-abelian interaction, Quarks

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## 1 Introduction

Quark-gluon are color charged particles. The dominant interaction among quarks and gluons are strong(color) interaction. Color interactions are analogous to electromagnetic interactions. Color charge particles produce chromoelectric field. Therefore chromomagnetic field will also be produced due to the motion of colored particles. Effect of chromomagnetism was earlier studied by Rujula, Georgi, and Glashow to explain mass splitting of hadron spectrum [1]. In this work they also predicted the existence of new states of hadrons with charm quarks and the mass of charm quarks. Chromomagnetism was also found useful to explain decay as well as production mechanisms of many hadrons [2, 3]. In these works various kind of chromomagnetic forces were found and were supported by experimental results.

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In a separate work Sapirstein examined the quark color magnetic moment in the framework of a quark with a constant external color magnetic field [4]. Ninomiya and Sakai studied the chromomagnetism to understand the phase transitions in quantum chromodynamics(QCD) [5]. Kapusta has studied the phase transition in a system of gluons interacting with a constant color magnetic field [6]. Therefore we find that chromomagnetism has played a vital role in explaining some complicated problems of QCD.

Color charge on quarks and gluons are responsible for strong or non-abelian interactions. Because of color charges they may produce chromoelectric and chromomagnetic fields. The chromoelectrodynamics of these particles is governed by the Yang-Mills equations which are analogs of the Maxwell's equations. As we know that hadrons are color neutral particles but with the help of string model we have proved that hadrons can have color magnetic moment. Because of color magnetic moment hadrons may show color interactions. In section two we have calculated the color magnetic moments of mesons and baryons separately. Then we have calculated the partition function. With the help of partition function we have studied the chromomagnetic properties of nuclear matter and possibilities of phase transition.

## 2 Formalism

Quarks and gluons are color charged particles. Due to color charge and their motion they can produce chromoelectric and chromomagnetic fields. In this paper we have discussed only quarks' contribution in chromomagnetism. Gluonic contributions will be discussed later in some other work.

Hadrons are color neutral particles but they can have color magnetic moment like a neutron, which is electrically neutral particle but it has non-zero magnetic moment. Therefore hadrons can have color interaction due to their color magnetic moment. Hence we have first estimated the chromomagnetic moment of hadrons. We have done the formulation in the framework of the string model of hadrons. According to this quark(or antiquark) are attached at the ends of the string. The string rotates about its center of mass. Due to rotation color currents are generated. These color currents are of two type: (i) abelian, and (ii) non-abelian. The abelian current is generated due to the orbital motion of quarks and non-abelian current is generated due to the temporal variation of color charge which is well described by the Wong equation [7]. The Wong's equation is written as

$$\frac{dQ^a}{d\tau} = f^{abc} u_\mu Q^b A^{c\mu} \quad (1)$$

where  $A^{a\mu}$  is the gauge potential, and  $f^{abc}$  is the structure constant of the gauge group,  $Q^a$  is the color charge of a quark, and  $a$  is the color index.  $u_\mu$  are four velocity vectors given by  $u_\mu = \gamma(1, -\mathbf{v})$ . It should be noted that in eq1 the derivative is wrt  $\tau$ (proper time) not wrt  $t$ . Therefore the color current

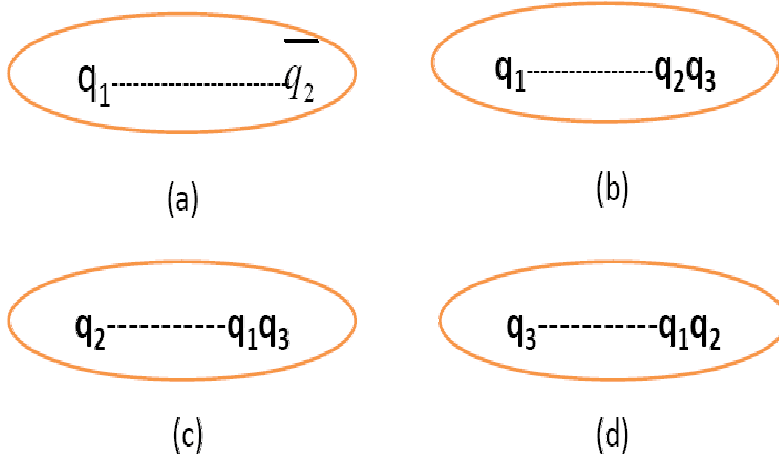
produced by a quark/antiquark is given by

$$I_{total}^a = I_{abelian}^a + I_{non-abelian}^a \quad (2)$$

With the help of Wong equation the color current is obtained as

$$I_{total}^a = \frac{Q^a \omega}{2\pi} + \frac{f^{abc}}{\gamma} Q^b u_\mu A^{c\mu} \quad (3)$$

where  $\omega$  is the angular velocity of quarks and  $\gamma$  is the relativistic factor. Due to these currents hadrons have intrinsic color magnetic moments.



**Fig. 1** (a) string model of meson; Fig (b),(c),(d): string model of baryon with different but equally probable configurations.

We have divided our analysis into two parts: (i) mesons & (ii) baryons.

### 2.1 Color magnetic moment of mesons

A meson contains quark and antiquark pair at the end of flux string (see fig a). Let us consider a general case of meson which contains two different kinds of quark-antiquark. Due to color neutrality of meson quark-antiquark pair will have equal and opposite color charges. The orbital color magnetic moment ( $\mu_1^a$ ) associated with quark '1' is given by

$$\mu_1^a = \left[ \frac{Q^a \omega}{2\pi} + f^{abc} Q^b \{A_o^c(\mathbf{r}_1) - \mathbf{v}_1 \cdot \mathbf{A}^c(\mathbf{r}_1)\} \right] \frac{\pi l^2 m_2^2}{(m_1 + m_2)^2} \hat{n} \quad (4)$$

where  $Q^a$ ,  $m_1$  are color charge and mass of quark '1',  $A_o^c$  &  $\mathbf{A}^c$  are gauge fields at  $\mathbf{r}_1$ . It is assumed that the quark  $q_1$  is situated at location  $\mathbf{r}_1$  wrt center of

mass of meson.  $f^{abc}$  is the structure constant of SU(3) group.  $m_2$  is the mass of antiquark '2'.  $l$  is the string length and  $\omega$  is the orbital angular velocity of quark.  $\hat{n}$  is the unit vector in the direction of the area sweep out by the rotational motion of string. Similarly we can calculate the orbital color magnetic moment of antiquark '2'. The net color magnetic moment of a meson is obtained by summing up the color magnetic moments produced by quark as well as antiquark. Therefore the net color magnetic moment of a meson is given by

$$\begin{aligned} \boldsymbol{\mu}^a = & \frac{1}{2} Q^a l^2 \omega \frac{m_2 - m_1}{m_2 + m_1} \hat{n} + [f^{abc} Q^b \{A_o^c(\mathbf{r}_1) - \mathbf{v}_1 \cdot \mathbf{A}^c(\mathbf{r}_1)\} m_2^2 \\ & - f^{ade} Q^d \{A_o^e(\mathbf{r}_2) - \mathbf{v}_2 \cdot \mathbf{A}^e(\mathbf{r}_2)\} m_1^2] \frac{\pi l^2}{(m_2 + m_1)^2} \hat{n} \end{aligned} \quad (5)$$

According to eq5 the color magnetic moment of a meson will have abelian as well as non-abelian components. Both abelian and non-abelian components are functions of masses of quarks. For a meson with same kind of quark/antiquark (like  $c\bar{c}$ ,  $b\bar{b}$  etc) the abelian component of color magnetic moment will be zero.

## 2.2 Color magnetic moment of baryons

A baryon consists of three quarks (say  $q_1, q_2, q_3$ ). It can have three possible configurations described by fig 1. b,c,& d. On repeating the analysis of mesons we calculate the color magnetic moments of all these configurations. The color magnetic moment for configuration described in fig1.b is given by

$$\begin{aligned} \boldsymbol{\mu}^{(1b)a} = & \frac{1}{2} l^2 \omega Q_1^a \frac{m_2 + m_3 - m_1}{m_1 + m_2 + m_3} \hat{n} + [f^{abc} Q_1^b \{A_o^c(\mathbf{r}_1) - \mathbf{v}_1 \cdot \mathbf{A}^c(\mathbf{r}_1)\} (m_2 + m_3)^2 \\ & - f^{ade} Q_1^d \{A_o^e(\mathbf{r}_2) - \mathbf{v}_2 \cdot \mathbf{A}^e(\mathbf{r}_2)\} m_1^2] \frac{\pi l^2}{(m_1 + m_2 + m_3)^2} \hat{n} \end{aligned} \quad (6)$$

Similarly color magnetic moments for other two possible configurations can be calculated. All these configurations are equally probable. Their average will be the color magnetic moment of a baryon. Therefore the color magnetic moment of a baryon is given by

$$\begin{aligned} \boldsymbol{\mu}^a = & \frac{1}{6} \frac{l^2 \omega}{m_1 + m_2 + m_3} [Q_1^a (m_2 + m_3 - m_1) + Q_2^a (m_1 + m_3 - m_2) \\ & + Q_3^a (m_1 + m_2 - m_3)] \hat{n} + [[f^{abc} Q_1^b \{A_o^c(\mathbf{r}_1) - \mathbf{v}_1 \cdot \mathbf{A}^c(\mathbf{r}_1)\} (m_2 + m_3)^2 \\ & - f^{ade} Q_1^d \{A_o^e(\mathbf{r}_2) - \mathbf{v}_2 \cdot \mathbf{A}^e(\mathbf{r}_2)\} m_1^2] \\ & + [f^{ab'c'} Q_2^{b'} \{A_o^{c'}(\mathbf{r}_1) - \mathbf{v}_1 \cdot \mathbf{A}^{c'}(\mathbf{r}_1)\} (m_1 + m_3)^2 \\ & - f^{ad'e'} Q_2^{d'} \{A_o^{e'}(\mathbf{r}_2) - \mathbf{v}_2 \cdot \mathbf{A}^{e'}(\mathbf{r}_2)\} m_2^2] \\ & + [f^{ab''c''} Q_3^{b''} \{A_o^{c''}(\mathbf{r}_1) - \mathbf{v}_1 \cdot \mathbf{A}^{c''}(\mathbf{r}_1)\} (m_1 + m_2)^2 \\ & - f^{ad''e''} Q_3^{d''} \{A_o^{e''}(\mathbf{r}_2) - \mathbf{v}_2 \cdot \mathbf{A}^{e''}(\mathbf{r}_2)\} m_3^2] \frac{\pi l^2}{(m_1 + m_2 + m_3)^2} \hat{n} \end{aligned} \quad (7)$$

From the above expression it is clear that the color magnetic moment of baryons also have abelian as well as non-abelian components.

### 2.3 Chromomagnetic properties of nuclear matter

Let us consider a sample of nuclear matter where hadrons (irrespective of baryon or meson) do not interact among themselves. To study the thermodynamic properties we first calculate the partition function. The partition function for such system is given by

$$z = \exp\{-\beta(\epsilon - \boldsymbol{\mu}^a \cdot \mathbf{B}^a)\} + \exp\{-\beta(\epsilon + \boldsymbol{\mu}^a \cdot \mathbf{B}^a)\} \quad (8)$$

where  $\epsilon$  is the non-magnetic energy associated with a hadron,  $\beta$  is the temperature inverse of hadronic system, and  $\mathbf{B}^a$  is the external color magnetic field acting on the nuclear matter. Therefore the mean color magnetisation (color magnetic moment per unit volume),  $\mathbf{M}^a$ , of a hadronic medium is given by

$$\mathbf{M}^a = C\mu^a \tanh(\beta\mu^b B^b) \quad (9)$$

where  $C$  is number density of hadron for a stable nuclear matter but for unstable quark matter like quark-gluon plasma (QGP) there will be continuous production of hadrons then  $C$  will indicate hadron production rate and obviously  $\mathbf{M}^a$  will show the rate of change of magnetization of hadronic matter.

At very large temperature

$$\mathbf{M}^a = C \frac{\mu^a \mu^b B^b}{k_B T} \quad (10)$$

On following the works of Karsch [8] and Huang and Lissia [9] one can show

$$Q^2(T) = 4\pi / \left[ 9 \ln \left( \frac{T}{0.1254 T_c} \right)^2 \right] \quad (11)$$

for three quark flavors. In eq11  $T_c$  is the QCD phase transition temperature. In other words the color charge of a quark is a temperature dependent quantity. Therefore from equations 5,7, 10, and 11 it is clear that the color magnetism of hadronic matter does not follow the Curie law. Due to the presence of non-linear temperature dependence and non-abelian component of color magnetic moment it will show complicated behavior.

### 3 Conclusions

So far we have studied chromomagnetism produced by quarks only. However hadrons are color neutral particles. But they may have non-zero chromomagnetic moment. Due to this chromomagnetic moment they will show color interactions among each other. Due to the non-abelian nature of strong interactions there are two components of chromomagnetic moment of hadrons: (i) abelian

and (ii) non-abelian. This may cause the existence of different chromomagnetic phases of nuclear matter. From the analysis for a non-interacting hadronic system we have seen that chromomagnetism of hadronic matter does not follow the Curie law.

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